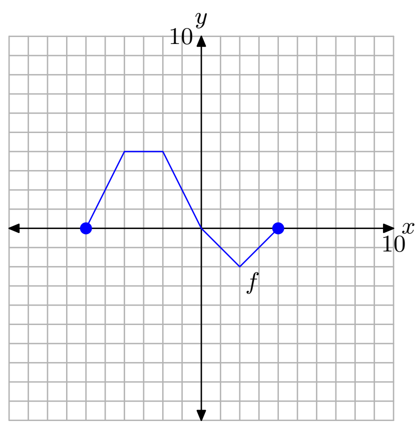
2.6 Exercises

Pictured below is the graph of a function f.



The table that follows evaluates the function f in the plot at key values of x. Notice the horizontal format, where the first point in the table is the ordered pair $(-6, 0)$.

x	-6	-4	-2	0	2	4
$f(x)$	0	4	4	0	-2	0

Use the graph and the table to complete each of following tasks for **Exercises 1-10**.

i. Set up a coordinate system on graph paper. Label and scale each axis, then copy and label the original graph of f onto your coordinate system. *Remember to draw all lines with a ruler.*
ii. Use the original table to help complete the table for the given function in the exercise.
iii. Using a different colored pencil, plot the data from your completed table on the *same* coordinate system as the original graph of f. Use these points to help complete the graph of the given function in the exercise, then label this graph with its equation given in the exercise.

1. $y = f(2x)$.

x	-3	-2	-1	0	1	2
y						

2. $y = f((1/2)x)$.

x	-12	-8	-4	0	4	8
y						

3. $y = f(-x)$.

x	-4	-2	0	2	4	6
y						

4. $y = f(x + 3)$.

x	-9	-7	-5	-3	-1	1
y						

5. $y = f(x - 1)$.

x	-5	-3	-1	1	3	5
y						

6. $y = f(-2x)$.

x	-2	-1	0	1	2	3
y						

7. $y = f((-1/2)x)$.

x	-8	-4	0	4	8	12
y						

8. $y = f(-x-2)$.

x	-6	-4	-2	0	2	4
y						

9. $y = f(-x+1)$.

x	-3	-1	1	3	5	7
y						

10. $y = f(-x/4)$.

x	-16	-8	0	8	16	24
y						

11. Use your graphing calculator to draw the graph of $y = \sqrt{x}$. Then, draw the graph of $y = \sqrt{-x}$. In your own words, explain what you learned from this exercise.

12. Use your graphing calculator to draw the graph of $y = |x|$. Then, draw the graph of $y = |-x|$. In your own words, explain what you learned from this exercise.

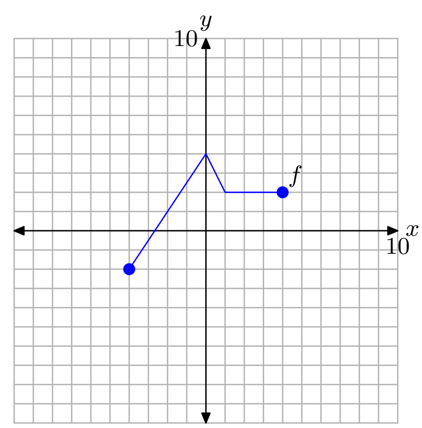
13. Use your graphing calculator to draw the graph of $y = x^2$. Then, in succession, draw the graphs of $y = (x-2)^2$, $y = (x-4)^2$, and $y = (x-6)^2$. In your own words, explain what you learned from this exercise.

14. Use your graphing calculator to draw the graph of $y = x^2$. Then, in succession, draw the graphs of $y = (x+2)^2$, $y = (x+4)^2$, and $y = (x+6)^2$. In your own words, explain what you learned from this exercise.

15. Use your graphing calculator to draw the graph of $y = |x|$. Then, in succession, draw the graphs of $y = |2x|$, $y = |3x|$, and $y = |4x|$. In your own words, explain what you learned from this exercise.

16. Use your graphing calculator to draw the graph of $y = |x|$. Then, in succession, draw the graphs of $y = |(1/2)x|$, $y = |(1/3)x|$, and $y = |(1/4)x|$. In your own words, explain what you learned from this exercise.

Pictured below is the graph of a function f. In **Exercises 17-22**, use this graph to perform each of the following tasks.



i. Set up a coordinate system on a sheet of graph paper. Label and scale each axis. Make an exact copy of the graph of f on your coordinate system. *Remember to draw all lines with a ruler.*
ii. In the narrative, a shadow box at the end of the section summarizes the concepts and technique of horizontal scaling, horizontal reflection, and horizontal translation. Use the shortcut ideas presented in this summary shadow

box to draw the graphs of the functions that follow **without** using tables.

iii. Use a different colored pencil to draw the graph of the function given in the exercise. Label this graph with its equation. Be sure that key points are accurately plotted. In each exercise, please plot exactly two plots per coordinate system, the graph of original function f and the graph of the function in the exercise.

17. $y = f(2x)$.

18. $y = f((1/2)x)$.

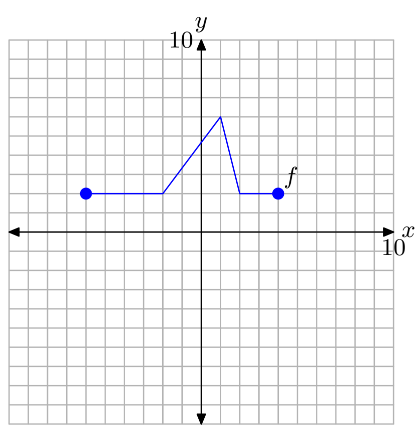
19. $y = f(-x)$.

20. $y = f(x-1)$.

21. $y = f(x+3)$.

22. $y = f(x-2)$.

Pictured below is the graph of a function f. In **Exercises 23-28**, use this graph to perform each of the following tasks.



i. Set up a coordinate system on a sheet of graph paper. Label and scale each axis. Make an exact copy of the graph of f on your coordinate system. *Remember to draw all lines with a ruler.*

ii. In the narrative, a shadow box at the end of the section summarizes the concepts and technique of horizontal scaling, horizontal reflection, and horizontal translation. Use the shortcut ideas presented in this summary shadow box to draw the graphs of the functions that follow **without** using tables.

iii. Use a different colored pencil to draw the graph of the function given in the exercise. Label this graph with its equation. Be sure that key points are accurately plotted. In each exercise, please plot exactly two plots per coordinate system, the graph of original function f and the graph of the function in the exercise.

23. $y = f(2x)$.

24. $y = f((1/2)x)$.

25. $y = f(-x)$.

26. $y = f(x+3)$.

27. $y = f(x-2)$.

28. $y = f(x+1)$.

2.6 Solutions

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1. The original function table.

x	-6	-4	-2	0	2	4
$f(x)$	0	4	4	0	-2	0

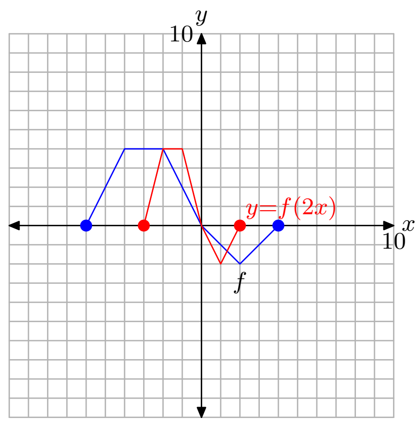
Evaluate the function $y = f(2x)$ at $x = -3$, -2, -1, 0, 1, and 2.

$$y = f(2(-3)) = f(-6) = 0$$
$$y = f(2(-2)) = f(-4) = 4$$
$$y = f(2(-1)) = f(-2) = 4$$
$$y = f(2(0)) = f(0) = 0$$
$$y = f(2(1)) = f(2) = -2$$
$$y = f(2(2)) = f(4) = 0$$

Points satisfying $y = f(2x)$.

x	-3	-2	-1	0	1	2
y	0	4	4	0	-2	0

Plot the points in the table to get the graph of $y = f(2x)$.



Note that replacing x with $2x$, as in $y = f(2x)$, compresses the graph of $y = f(x)$ horizontally by a factor of 2.

3. The original function table.

x	-6	-4	-2	0	2	4
$f(x)$	0	4	4	0	-2	0

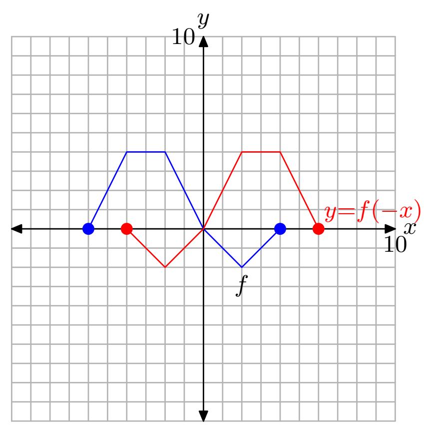
Evaluate the function $y = f(-x)$ at $x = -4$, -2, 0, 2, 4, and 6.

$$\begin{aligned}
y &= f(-(-4)) = f(4) = 0 \\
y &= f(-(-2)) = f(2) = -2 \\
y &= f(-(0)) = f(0) = 0 \\
y &= f(-(2)) = f(-2) = 4 \\
y &= f(-(4)) = f(-4) = 4 \\
y &= f(-(6)) = f(-6) = 0
\end{aligned}$$

Points satisfying $y = f(-x)$.

x	-4	-2	0	2	4	6
y	0	-2	0	4	4	0

Plot the points in the table to get the graph of $y = f(-x)$.



Note that replacing x with $-x$, as in $y = f(-x)$, reflects the graph of $y = f(x)$ across the y-axis.

5. The original function table.

x	-6	-4	-2	0	2	4
$f(x)$	0	4	4	0	-2	0

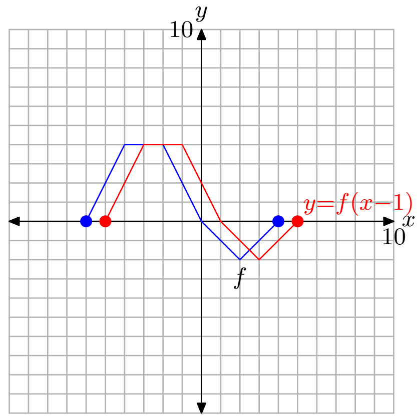
Evaluate the function $y = f(x - 1)$ at $x = -5$, -3, -1, 1, 3, and 5.

$$\begin{aligned}
y &= f((-5) - 1) = f(-6) = 0 \\
y &= f((-3) - 1) = f(-4) = 4 \\
y &= f((-1) - 1) = f(-2) = 4 \\
y &= f((1) - 1) = f(0) = 0 \\
y &= f((3) - 1) = f(2) = -2 \\
y &= f((5) - 1) = f(4) = 0
\end{aligned}$$

Points satisfying $y = f(x - 1)$.

x	-5	-3	-1	1	3	5
y	0	4	4	0	-2	0

Plot the points in the table to get the graph of $y = f(x - 1)$.



Note that replacing x with $x - 1$, as in $y = f(x - 1)$, translates the graph of $y = f(x)$ horizontally 1 unit to the right.

7. The original function table.

x	-6	-4	-2	0	2	4
$f(x)$	0	4	4	0	-2	0

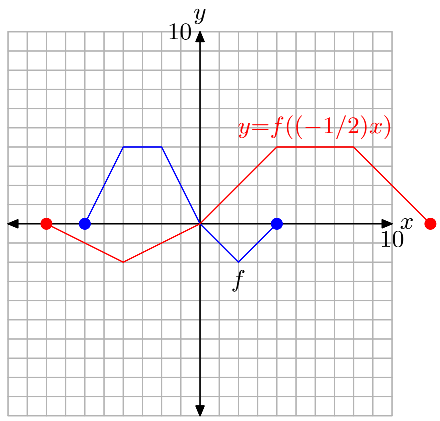
Evaluate the function $y = f((-1/2)x)$ at $x = -8$, -4, 0, 4, 8, and 12.

$$y = f((-1/2)(-8)) = f(4) = 0$$
$$y = f((-1/2)(-4)) = f(2) = -2$$
$$y = f((-1/2)(0)) = f(0) = 0$$
$$y = f((-1/2)(4)) = f(-2) = 4$$
$$y = f((-1/2)(8)) = f(-4) = 4$$
$$y = f((-1/2)(12)) = f(-6) = 0$$

Points satisfying $y = f((-1/2)x)$.

x	-8	-4	0	4	8	12
y	0	-2	0	4	4	0

Plot the points in the table to get the graph of $y = f((-1/2)x)$.



Note that replacing x with $(-1/2)x$, as in $y = f((-1/2)x)$, stretches the graph by a factor of 2, then reflects the result across the y-axis.

9. The original function table.

x	-6	-4	-2	0	2	4
$f(x)$	0	4	4	0	-2	0

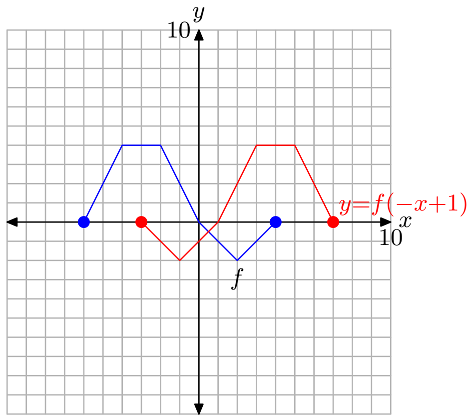
Evaluate the function $y = f(-x+1)$ at $x = -3$, -1, 1, 3, 5, and 7.

$$\begin{aligned}
y &= f(-(-3)+1) = f(4) = 0\\
y &= f(-(-1)+1) = f(2) = -2\\
y &= f(-(1)+1) = f(0) = 0\\
y &= f(-(3)+1) = f(-2) = 4\\
y &= f(-(5)+1) = f(-4) = 4\\
y &= f(-(7)+1) = f(-6) = 0
\end{aligned}$$

Points satisfying $y = f(-x+1)$.

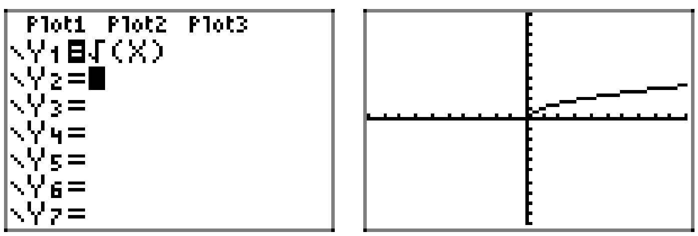
x	-3	-1	1	3	5	7
y	0	-2	0	4	4	0

Plot the points in the table to get the graph of $y = f(-x+1)$.



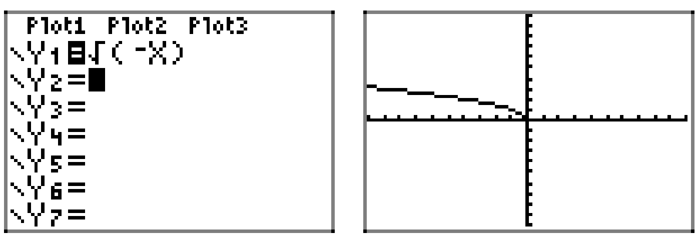
Note that $y = f(-x+1)$ is the same as $y = f(-(x-1))$. If we replace x with $-x$ to get $y = f(-x)$, then x in this last result with $x-1$ to get $y = f(-(x-1))$, this has the effect of first reflecting the graph of $y = f(x)$ across the y-axis, then shifting the result to the right 1 unit.

11. First, draw the graph of $y = \sqrt{x}$.



(a) (b)

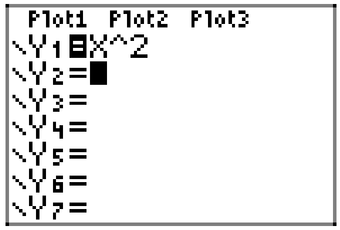
The graph of $y = \sqrt{-x}$ is a reflection of the graph of $y = \sqrt{x}$ across the y-axis.



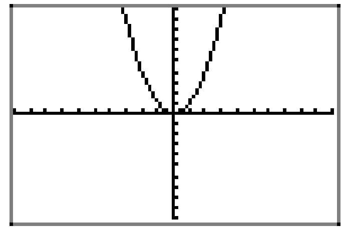
(c) (d)

Replacing x with $-x$, as in $y = f(-x)$, reflects the graph of $y = f(x)$ across the y-axis.

13. First, draw the graph of $y = x^2$.

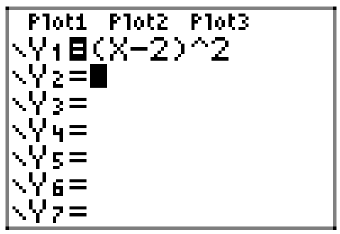


(a)

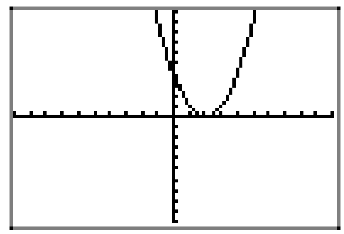


(b)

Replacing x with $x - 2$ translates the graph of $y = x^2$ two units to the right in the horizontal direction.

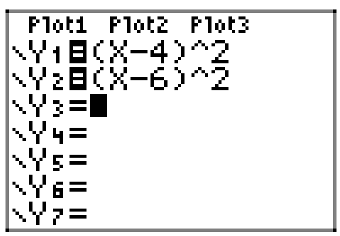


(c)

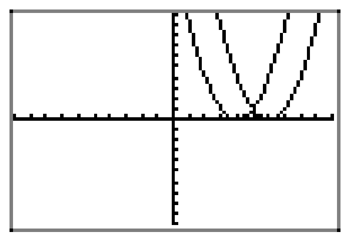


(d)

Similarly, replacing x with $x - 4$ and $x - 6$ translates the graph of $y = x^2$ four units and 6 units to the right, respectively.



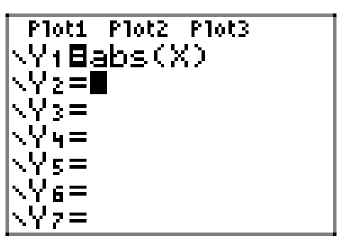
(e)



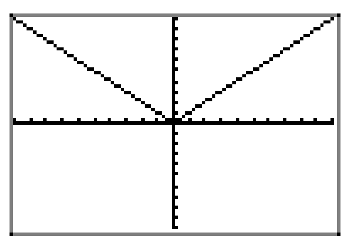
(f)

In general, if c is positive, then the graph of $y = f(x - c)$ is obtained by translating the graph of $y = f(x)$ to the right c units.

15. First, draw the graph of $y = |x|$.

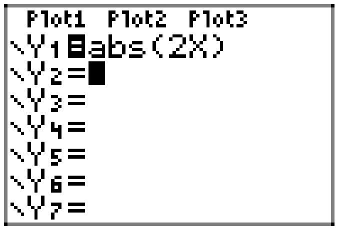


(a)

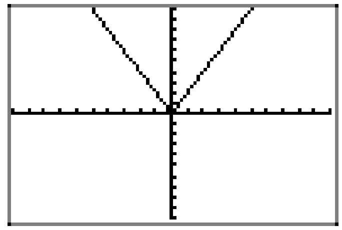


(b)

Replacing x with $2x$ compresses the graph of $y = |x|$ by a factor of 2 in the horizontal direction.

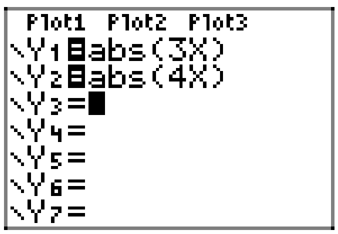


(c)

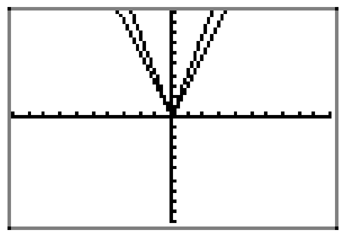


(d)

Similarly, replacing x with $3x$ and $4x$ by a factor of 3 and 4 in the horizontal direction, respectively.



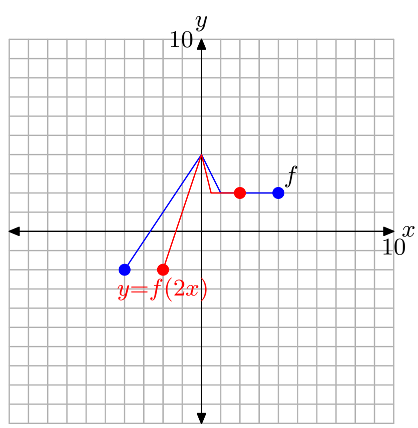
(e)



(f)

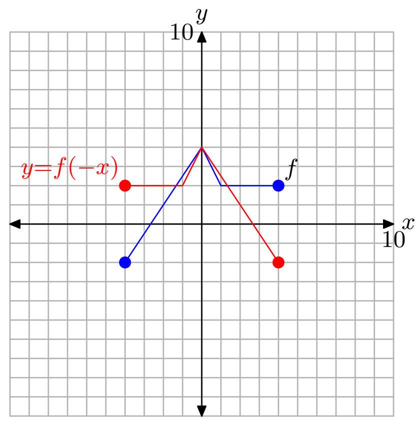
In general, if $a > 1$, then the graph of $y = f(ax)$ is obtained by compressing the graph of $y = f(x)$ by a factor of a in the horizontal direction.

17. To obtain a plot for $y = f(2x)$, take each point on the graph of $y = f(x)$ and divide its x-value by 2, keeping the y-value the same.



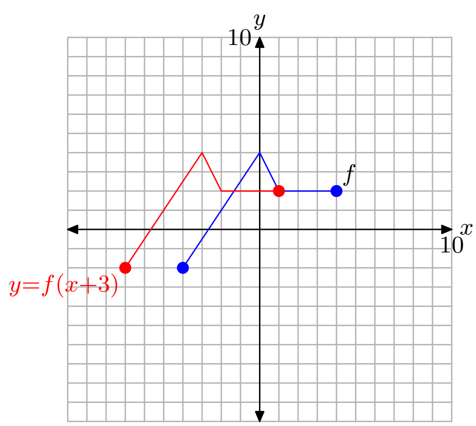
Note that replacing x with $2x$, as in $y = f(2x)$, compresses the graph of $y = f(x)$ in the horizontal direction by a factor of 2.

19. To obtain a plot of $y = f(-x)$, take each point on the graph of $y = f(x)$ and negate its x-value, keeping the y-value the same.



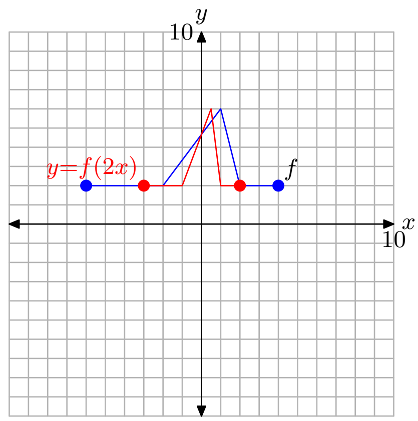
Note that replacing x with $-x$, as in $y = f(-x)$, reflects the graph of f across the y-axis.

21. To obtain a plot of $y = f(x+3)$, take each point on the graph of $y = f(x)$ and subtract 3 from its x-value, keeping the y-value the same.



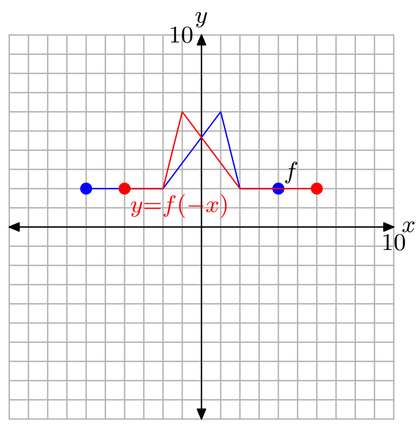
Note that replacing x with $x+3$, as in $y = f(x+3)$, translates the graph of $y = f(x)$ to the left 3 units.

23. To obtain a plot of $y = f(2x)$, take each point on the graph of $y = f(x)$ and divide its x-value by 2, keeping the y-value the same.



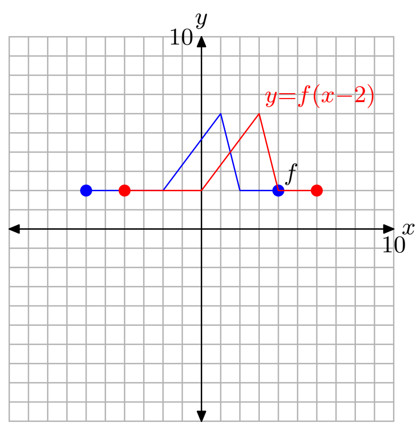
Replacing x with $2x$, as in $y = f(2x)$, compresses the graph of $y = f(x)$ horizontally by a factor of 2.

25. To obtain a plot of $y = f(-x)$, take each point on the graph of $y = f(x)$ and negate its x-value, keeping the same y-value.



Replacing x with $-x$, as in $y = f(-x)$, reflects the graph of $y = f(x)$ across the y-axis.

27. To obtain a plot of $y = f(x - 2)$, take each point on the graph of $y = f(x)$ and add 2 to its x-value, keeping its y-value the same.



Replacing x with $x - 2$, as in $y = f(x - 2)$, shifts the graph of $y = f(x)$ to the right 2 units.