Pizza and Problems

Fall 2007

Assigned on: October 26, 2007 Due on: October 26, 2007

PROBLEM 1 Given a set of n numbers, of which one of the numbers is $1 - (1/n)$ and the others are all 1, find the arithmetic mean.

PROBLEM 2 A regular dodecagon in inscribed in a circle of radius r inches. Find the area of the dodecagon.

PROBLEM 3 A polynomial $p(x)$ has remainder 3 when divided by $x - 1$ and remainder 5 when divided by $x - 3$. Find the remainder when $p(x)$ is divided by $(x - 1)(x - 3)$.

PROBLEM 4 Find the difference between the roots of $f(x) = x^2 - px + (p^2 - 1)/4$.

PROBLEM 5 A circle of radius r is inscribed in a right isosceles triangle, and a circle of radius R is circumscribed about the triangle. Compute the ratio R/r.

PROBLEM 6 Find the angle θ with $0 \leqslant \theta < 2\pi$ at which the expression

$$\cos(x/2) - \sqrt{3}\sin(x/2)$$

has its minimum value.

PROBLEM 7 A man on his way to dinner shortly after 6:00 pm observes that the hands of his watch form an angle of 110 degrees. Returning before 7:00 pm, he notices that again the hands of his watch form an angle of 110 degrees. (The watch is not broken and operates normally.) Find the number of minutes that he has been away.