Pizza and Problems

Fall 2007

Assigned on: November 9, 2007 Due on: November 9, 2007

PROBLEM 1 Allen takes m times as long to do a piece of work as Bob and Carl together; Bob takes n times as long as Carl and Allen together; and Carl takes x times as long as Allen and Bob together. Find x in terms of m and n.

PROBLEM 2 A man walks x miles due west, turns 150° to his left and walks 3 miles in the new direction. If he finishes at a point $\sqrt{3}$ miles from his starting point, what is the value of x?

PROBLEM 3 Straight lines OA and OB are distinct. From a point on OA, a perpendicular is drawn to OB; from the foot of this perpendicular, a line is drawn perpendicular to OA. From the foot of this second perpendicular, a line is drawn perpendicular to OB; and so on, indefinitely. The lengths of the first and second perpendiculars are a and b, respectively. The sum of the lengths of the perpendiculars approaches a limit as the number of perpendiculars grows beyond all bounds. Find the limit.

PROBLEM 4 The function f satisfies the equation

$$f(x) + f(y) = f(x + y) - xy - 1$$

for every pair x, y of real numbers. If $f(1) = 1$, find all other integers n (if any) such that $f(n) = n$.

PROBLEM 5 If $x^4 + 4x^3 + 6px^2 + 4qx + r$ is exactly divisible by $x^3 + 3x^2 + 9x + 3$, find the value of $(p + q)r$.

PROBLEM 6 Two identical jars are filled with alcohol solutions, and the ratio of the volume of alcohol to the volume of water is $p : 1$ in one jar and $q : 1$ in the other jar. If the entire contents of the two jars are mixed together, find the ratio of the volume of alcohol to the volume of water in the mixture.

PROBLEM 7 Compute the exact value of

$$(\log_2 3)(\log_3 4)(\log_4 5)\ldots(\log_{31} 32)$$

PROBLEM 8 Three fair dice are tossed. What is the probability that the three numbers turned up can be arranged to form an arithmetic progression?