Pizza and Problems

Fall 2007

Assigned on: September 28, 2007 Due on: September 28, 2007

PROBLEM 1 Two tangents are drawn to a circle from an exterior point A; they touch the circle at points B and C, respectively. A third tangent intersects segment AB in P and AC in R, and touches the circle at Q. If $AB = 20$, find the perimeter of triangle $\triangle APR$.

PROBLEM 2 In the base ten number system the number 526 means $5 \cdot 10^2 + 2 \cdot 10 + 6$. In the Land of Mathesis, however, numbers are written in the base r. James purchases an automobile there for 440 monetary units (abbreviated m.u.). He gives the salesman a 1000 m.u. bill, and receives, in change, 340 m.u. What is the base r?

PROBLEM 3 In racing over a given distance d at uniform speed, A can beat B by 20 yards, B can beat C by 10 yards, and A can beat C by 28 yards. What is the distance d in yards?

PROBLEM 4 Find the minimum value of $\sqrt{x^2 + y^2}$ if $5x + 12y = 60$.

PROBLEM 5 The population of Nosuch Junction at one time was a perfect square. Later, with an increase of 100, the population was one more than a perfect square. Now, with an additional increase of 100, the population is again a perfect square. The original population is a multiple of:

(a) 3

(b) 7

(c) 9

(d) 11

(e) 17

PROBLEM 6 If θ is a constant such that $0 < \theta < \pi$ and $x + \frac{1}{x} = 2\cos(\theta)$, then for each positive integer n, $x^n + \frac{1}{x^n} =$

(a) $2\cos(\theta)$

(b) $2^n \cos(\theta)$

(c) $2\cos^n(\theta)$

(d) $2\cos(n\theta)$

(e) $2^n \cos^n(\theta)$

PROBLEM 7 A bug (of negligible size) starts at the origin on the xy-plane. First it moves 1 unit right to $(1, 0)$. Then it makes a $90°$ turn counterclockwise and travels $1/2$ unit to $(1, 1/2)$. If it continues in this fashion, each time making a $90°$ turn counterclockwise and traveling half as far as in the previous move, to which of the following points will it come closest?

(a) $\left(\frac{2}{3}, \frac{3}{3}\right)$

(b) $\left((\frac{4}{5}, \frac{2}{5}\right)$

(c) $\left((\frac{2}{3}, \frac{4}{5}\right)$

(d) $\left((\frac{2}{3}, \frac{1}{3}\right)$

(e) $\left((\frac{2}{5}, \frac{4}{5}\right)$