

Pizza and Problems

Fall 2007

Assigned on: October 12, 2007

Due on: October 12, 2007

PROBLEM 1 If the line $y = mx + 1$ intersects the ellipse $x^2 + 4y^2 = 1$ exactly once, find the value of m^2 .

PROBLEM 2 Let f_1 be the function $f_1(x) = \frac{2x-1}{x+1}$, and define $f_{n+1}(x) = f_1(f_n(x))$ for $n = 1, 2, 3, \dots$. It can be shown that $f_{35} = f_5$. It then follows that $f_{28}(x) =$

- (a) x
- (b) $\frac{1}{x}$
- (c) $\frac{x-1}{x}$
- (d) $\frac{1}{1-x}$
- (e) none of these

PROBLEM 3 A square and an equilateral triangle have equal perimeters. The area of the triangle is $9\sqrt{3}$ square inches. Find the diagonal of the square.

PROBLEM 4 Try each of the following:

- (a) Find all positive integers n , such that $2^n - 1$ is divisible by 7.
- (b) Prove that there is no positive integer n such that $2^n + 1$ is divisible by 7.

PROBLEM 5 Determine all values x in the interval $0 \leq x \leq 2\pi$ which satisfy the inequality

$$2 \cos x \leq \left| \sqrt{1 + \sin 2x} - \sqrt{1 - \sin 2x} \right| \leq \sqrt{2}.$$

PROBLEM 6 If $a = \log_8 225$ and $b = \log_2 15$, then find a in terms of b .