

Pizza and Problems

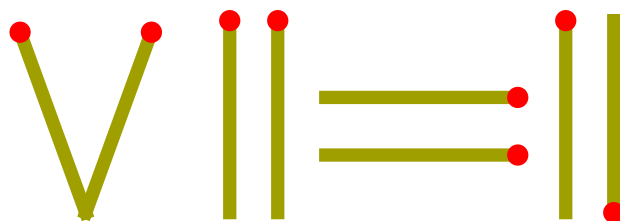
Fall 2008

Assigned on: September 12, 2008

Due on: September 12, 2008

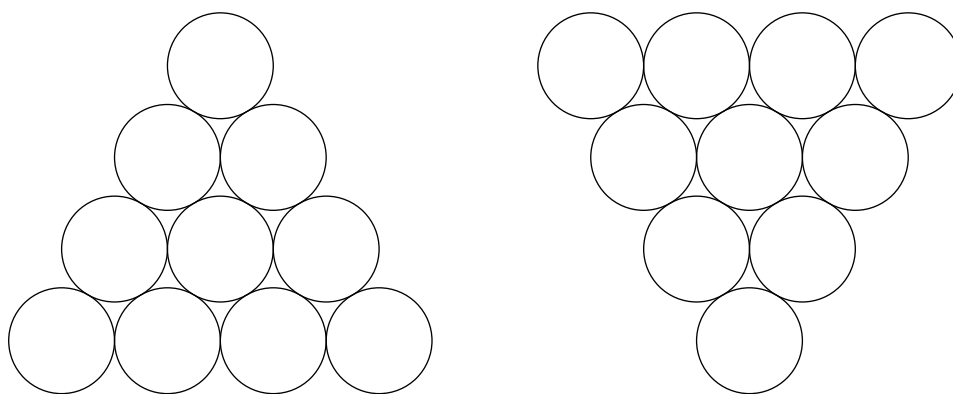
PROBLEM 1 A six-story house (not counting the basement) has stairs of the same length from floor to floor. How many times as high is a climb from the first to the sixth floor as a climb from the first to the third floor?

PROBLEM 2 Move one match in the following figure to produce a valid equation. Crossing the equality sign with a match to make it a not-equal sign is ruled out.



PROBLEM 3 Two real numbers x and y are chosen at random in the interval $(0, 1)$ with respect to the uniform distribution. What is the probability that the closest integer to x/y is even? Express the answer in the form $r + s\pi$, where r and s are rational numbers.

PROBLEM 4 Pack 10 pennies to form a triangle [at the left in the figure below]. This is the famous “tetractys” of the ancient Pythagoreans and today’s familiar pattern for the 10 bowling pins. The problem is to turn this triangle upside down [at the right in the figure below] by sliding one penny at a time to a new position in which it touches two other pennies. What is the minimum number of required moves?



PROBLEM 5 An oil well being drilled in flat prairie country struck pay sand at an underground spot exactly 21,000 feet from one corner of a rectangular plot of farmland, 18,000 feet from the opposite corner, and 6,000 feet from a third corner. How far is the underground spot from the

Assigned: September 12, 2008

1

Due: September 12, 2008

Pizza and Problems

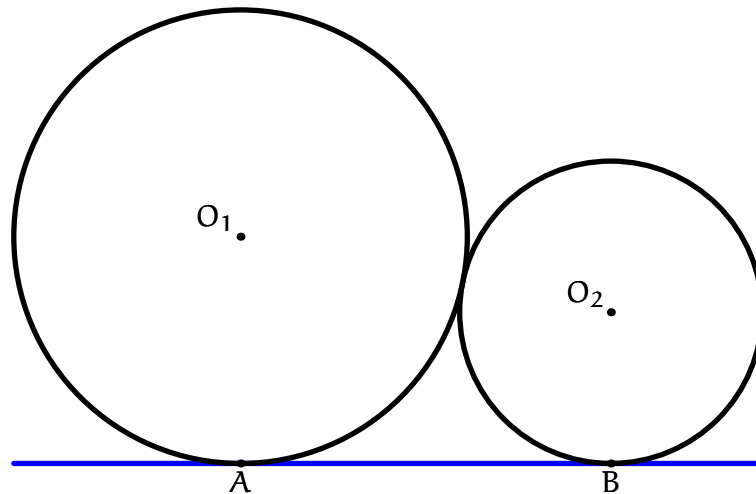
fourth corner? By solving the problem you will discover a useful formula of great generality and delightful simplicity.

PROBLEM 6 Each circle in an infinite sequence with decreasing radii is tangent externally to the one following it and to both sides of a given right angle. Find the ratio of the area of the first circle to the sum of areas of all other circles in the sequence.

PROBLEM 7 If $x = (1 - i\sqrt{3})/2$, where $i = \sqrt{-1}$, then simplify $1/(x^2 - x)$. *Note: You can crunch and grind this problem, but there is a very cute geometrical approach.*

PROBLEM 8 The number $N = 2^{12} \times 5^5$ has how many digits?

PROBLEM 9 Sangaku are Japanese geometrical puzzles in Euclidean geometry on wooden tablets created during the Edo period (1603-1867) by members of all social classes. Here is an example of this ancient geometrical art form. The circles centered at O_1 and O_2 have radii r_1 and r_2 , respectively. The circles touch at a single point and are tangent to the line shown at points A and B. Prove that $(AB)^2 = 4r_1r_2$. That is, show that the square of the distance between A and B is $4r_1r_2$.



1 Wiki Page

Our wiki page for Pizza and Problems is located at the following URL:

http://msenex.redwoods.edu/wiki/index.php/Pizza_and_Problems

If interested in editing solutions on this page, you need an account. If you wish an account, post an email to david-arnold@redwoods.edu that includes a username and password which you wish to use to log into the wiki.