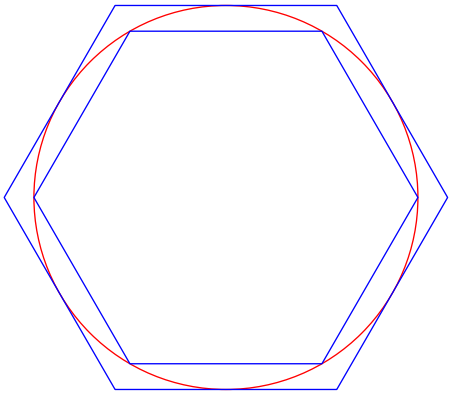
# *Pizza and Problems*

## Fall 2008

Assigned on: September 26, 2008 Due on: September 26, 2008

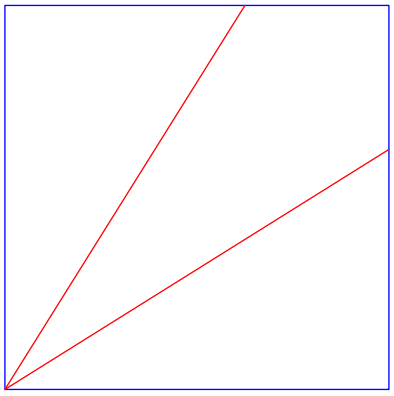
PROBLEM 1 In the figure that follows, regular hexagons are inscribed in and circumscribed outside a circle. If the smaller hexagon has an area of three square units, what is the area of the larger hexagon?



PROBLEM 2 Each of the two equal sides of an isosceles triangle is one unit long. Find the length of the third side that maximizes the triangle's area.

PROBLEM 3 A deck of 52 playing cards is shuffled and placed facedown on the table. Then, one at a time, the cards are dealt face up from the top. If you were asked to bet in advance on the distance from the top of the first black ace to be dealt, what position (first, second, third, ...) would you pick so that if the game were repeated many times, you would maximize your chance in the long run of guessing correctly?

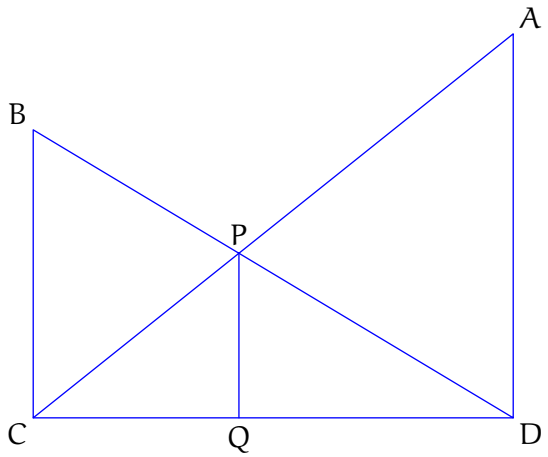
PROBLEM 4 From one corner of a square extend two lines that exactly trisect the square's area (see figure that follows). Into what ratios do these trisecting lines cut the two sides of the square?



PROBLEM 5 Find, with explanation, the maximum value of $f(x) = x^3 - 3x$ on the set of all real numbers $x$ satisfying $x^4 + 36 \leqslant 13x^2$.

PROBLEM 6 In quadrilateral ABCD (see figure below), AD is parallel to BC, AD and BC are perpendicular to CD, the diagonals AC and BD meet at P, and Q is the foot of the perpendicular from P to CD. Show that

$$\frac{1}{AD} + \frac{1}{BC} = \frac{1}{PQ}.$$




PROBLEM 7 The first term of an arithmetic series of consecutive integers is $k^2 + 1$. Find the sum of the first $2k + 1$ terms of this series.

PROBLEM 8 If $4^x - 4^{x-1} = 24$, then $(2x)^x$ equals?

PROBLEM 9 Two men set out at the same time to walk towards each other from M and N, 72 miles apart. The first man walks at a rate of 4 mph. The second man walks 2 miles the first hour, $2\frac{1}{2}$ miles the second hour, 3 miles the third hour, and so on in arithmetic progression. When and where will the men meet?

## 1 Wiki Page

Our wiki page for Pizza and Problems is located at the following URL:

http://msenux.redwoods.edu/wiki/index.php/Pizza_and_Problems

If interested in editing solutions on this page, you need an account. If you wish an account, post an email to david-arnold@redwoods.edu that includes a username and password which you wish to use to log into the wiki.