

# Pizza and Problems

Fall 2008

Assigned on: October 17, 2008

Due on: October 17, 2008

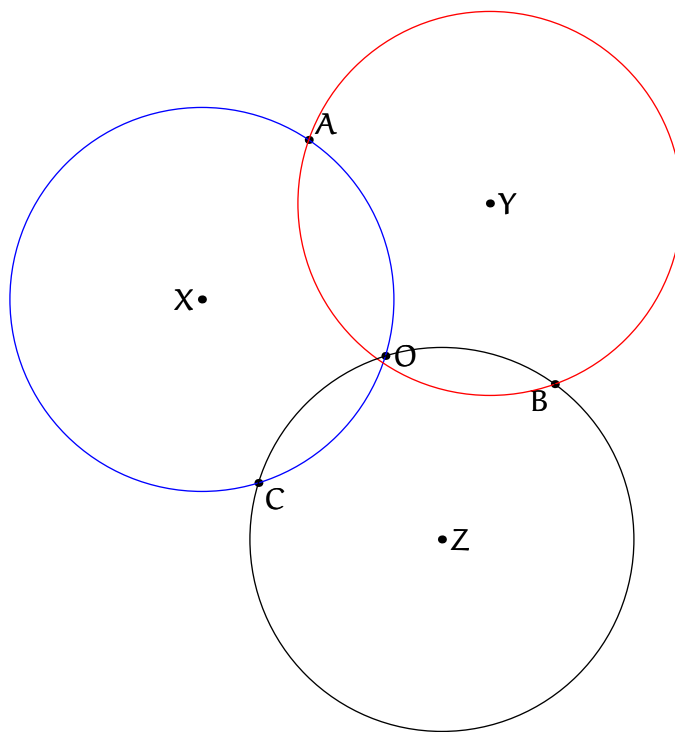
**PROBLEM 1** *The Square Root of Wonderful* was the name of a play on Broadway. If each letter in WONDERFUL stands for a different digit (zero excluded) and if OODDF, using the same code, represents the square root, then what is the square root of wonderful?

**PROBLEM 2** The number  $2^{2^9}$  is a 9-digit number with distinct digits. Which digit is missing?

**PROBLEM 3** What is the value of:

$$\frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \dots}}}$$

**PROBLEM 4** This is one of those elegant theorems in old-fashioned plane geometry that seem at first to be exceedingly difficult to establish but that yield readily to the right insight. Three circles of unit radius, with centers at X, Y, and Z, intersect at a common point, O. The problem is to prove that the other three intersection points, A, B, and C, lie on a circle that also has a unit radius.



Assigned: October 17, 2008

1

Due: October 17, 2008

## Pizza and Problems

**PROBLEM 5** An ellipse, whose semi-axes have lengths  $a$  and  $b$ , rolls without slipping on the curve  $y = c \sin(x/a)$ . How are  $a$ ,  $b$ , and  $c$  related, given that the ellipse completes one revolution when it traverses one period of the curve?

**PROBLEM 6** Determine all pairs  $(a, b)$  of positive integers such that  $ab^2 + b + 7$  divides  $a^2b + a + b$ .

**PROBLEM 7** My wife and I recently attended a party at which there were four other married couples. Various handshakes took place. No one shook hands with himself (or herself) or with his (or her) spouse, and no one shook hands with the same person more than once.

After all the handshakes were over, I asked each person, including my wife, how many hands he (or she) had shaken. To my surprise each gave a different answer. How many hands did my wife shake?

**PROBLEM 8** Three points are selected at random on a sphere's surface. What is the probability that all three lie on the same hemisphere? It is assumed that the great circle, bordering a hemisphere, is part of the hemisphere.

**PROBLEM 9** For the infinite series form

$$1 - \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} - \frac{1}{32} + \frac{1}{64} - \frac{1}{128} - \dots,$$

let  $S$  be the (limiting) sum. Then  $S$  equals what?

**PROBLEM 10** To each nonnegative integer  $n$ , we associate a new integer  $f(n)$ . Suppose that  $f(0) = 0$ ,  $f(1) = 1$ , and that for  $n \geq 2$  we have  $f(n) - 2f(n-1) + f(n-2) = (-1)^n(2n-4)$ . Describe  $f(n)$  in terms of  $n$ .

### 1 Wiki Page

Our wiki page for Pizza and Problems is located at the following URL:

[http://msenux.redwoods.edu/wiki/index.php/Pizza\\_and\\_Problems](http://msenux.redwoods.edu/wiki/index.php/Pizza_and_Problems)

If interested in editing solutions on this page, you need an account. If you wish an account, post an email to [david-arnold@redwoods.edu](mailto:david-arnold@redwoods.edu) that includes a username and password which you wish to use to log into the wiki.