

# Pizza and Problems

Fall 2008

Assigned on: November 14, 2008

Due on: November 14, 2008

**PROBLEM 1** A rectangle,  $HOMF$ , has sides  $HO = 11$  and  $OM = 5$ . A triangle  $ABC$  has  $H$  as the intersection of the altitudes,  $O$  the center of the circumscribed circle,  $M$  the midpoint of  $BC$ , and  $F$  the foot of the altitude from  $A$ . What is the length of  $BC$ ?

**PROBLEM 2** What is the ratio of the area of a square inscribed in a semicircle to the area of the square inscribed in the entire circle?

**PROBLEM 3** In base  $b$  the expanded fraction  $F_1$  becomes  $0.3737\dots = 0.\overline{37}$ , and the expanded fraction  $F_2$  becomes  $0.7373\dots = 0.\overline{73}$ . In base  $a$  the expanded fraction  $F_1$  becomes  $0.2525\dots = 0.\overline{25}$ , and the expanded fraction  $F_2$  becomes  $0.5252\dots = 0.\overline{52}$ . Find the sum of  $a$  and  $b$ , written in base ten.

**PROBLEM 4** The function  $f$  satisfies the functional equation

$$f(x) + f(y) = f(x + y) - xy - 1,$$

for every pair  $x, y$  of real numbers. If  $f(1) = 1$ , then how many integers  $n \neq 1$  exist such that  $f(n) = n$ ?

**PROBLEM 5** The letters of the word *ANNIVERSARY* can be arranged  $11!/(2!)^3 = 4,989,600$  different ways. When these "words" are arranged in alphabetical order, what is the tenth letter of the 50th word?

**PROBLEM 6** What is the value of the expression

$$\frac{1}{\log_2 100!} + \frac{1}{\log_3 100!} + \frac{1}{\log_4 100!} + \dots + \frac{1}{\log_{100} 100!}?$$

**PROBLEM 7** Find two positive integers  $x$  and  $y$  such that the product of their greatest common divisor and their lowest common multiple is  $xy$ .

**PROBLEM 8** What three positive integers have a sum equal to their product?

## 1 Wiki Page

Our wiki page for Pizza and Problems is located at the following URL:

[http://msenux.redwoods.edu/wiki/index.php/Pizza\\_and\\_Problems](http://msenux.redwoods.edu/wiki/index.php/Pizza_and_Problems)

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## Pizza and Problems

If interested in editing solutions on this page, you need an account. If you wish an account, post an email to david-arnold@redwoods.edu that includes a username and password which you wish to use to log into the wiki.<sup>1</sup>

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<sup>1</sup>Some problems on this page are extracted from a wonderful text by Martin Gardner, *The Collosal Book of Short Puzzles and Problems*, Norton Publishing. Other problems come from past Putnam and Math Olympiad examinations, and some come from Contest Problem Books published by the American Mathematical Society.