

Pizza and Problems

Fall 2008

Assigned on: December 5, 2008

Due on: December 5, 2008

PROBLEM 1 A game starts with four heaps of beans, containing 3, 4, 5, and 6 beans. A move consist of taking either

- (a) one bean from a heap, provided at least two beans are left behind in that heap, or
- (b) a complete heap of two or three beans.

The player who takes the last heap wins. To win the game, do you want to move first or second? Give a winning strategy.

PROBLEM 2 Define a sequence $\{a_i\}$ by $a_1 = 3$ and $a_{i+1} = 3^{a_i}$ for $i \geq 1$. Which integers between 00 and 99 inclusive occur as the last two digits in the decimal expansion of infinitely many a_i ?

PROBLEM 3 Evaluate

$$\int_0^{\infty} t^{-1/2} e^{-1985(t+t^{-1})} dt.$$

You may assume that

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}.$$

PROBLEM 4 Let I be the incenter of triangle ABC . Let the incircle of ABC touch the sides BC , CA , and AB at K , L , and M , respectively. The line through B parallel to MK meets the lines LM and LK at R and S , respectively. Prove that angle RIS is acute.

PROBLEM 5 The function $f(n)$ is defined for all positive integers n and takes on non-negative integer values. Also, for all m, n ,

$$f(m+n) - f(m) - f(n) = 0 \quad \text{or} \quad 1$$

and

$$f(2) = 0, \quad f(3) > 0, \quad \text{and} \quad f(9999) = 3333.$$

Determine $f(1982)$.

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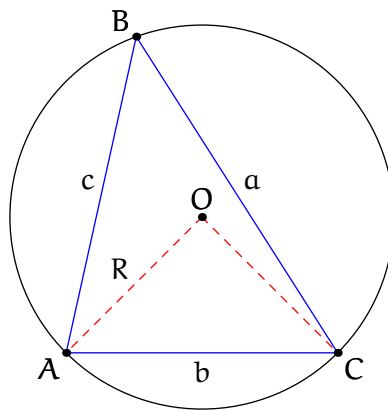
PROBLEM 6 The diagonals AC and CE of the regular hexagon ABCDEF are divided by the inner points M and N, respectively, so that

$$\frac{AM}{AC} = \frac{CN}{CE} = r.$$

Determine r if B, M, and N are collinear.

PROBLEM 7 Prove that the medians of a triangle meet in a point that divides each of the medians in a ratio of 2:1.

PROBLEM 8 The following shows the circumscribed circle for triangle ABC. The point O is the center of the circle and R is the radius.



Prove that

$$R = \frac{a}{2 \sin A} = \frac{b}{2 \sin B} = \frac{c}{2 \sin C}.$$

1 Wiki Page

Our wiki page for Pizza and Problems is located at the following URL:

http://msenux.redwoods.edu/wiki/index.php/Pizza_and_Problems

If interested in editing solutions on this page, you need an account. If you wish an account, post an email to david-arnold@redwoods.edu that includes a username and password which you wish to use to log into the wiki.¹

¹Some problems on this page are extracted from a wonderful text by Martin Gardner, *The Colossal Book of Short Puzzles and Problems*, Norton Publishing. Other problems come from past Putnam and Math Olympiad examinations, and some come from Contest Problem Books published by the American Mathematical Society.