

Pizza and Problems

Fall 2009

Assigned on: September 30, 2009

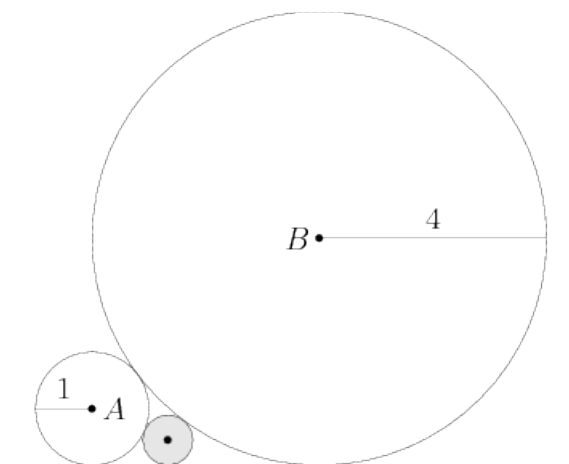
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PROBLEM 1 Let f be a function satisfying

$$f(xy) = \frac{f(x)}{y}$$

for all positive real numbers x and y . If $f(500) = 3$, what is the value of $f(600)$?

PROBLEM 2 A circle centered at A with a radius of 1 and a circle centered at B with a radius of 4 are externally tangent. A third circle is tangent to the first two and to one of their common external tangents as shown. What is the radius of the third circle?



PROBLEM 3 For positive integers a , b , c , and d have a product $8!$ and satisfy

$$ab + a + b = 524$$

$$bc + b + c = 146$$

$$cd + c + d = 104$$

What is $a - d$?

PROBLEM 4 A polynomial of degree four with leading coefficient 1 and integer coefficients has two zeros, both of which are integers. Which of the following can also be a zero of the polynomial?

- (a) $\frac{1 + i\sqrt{11}}{2}$ (b) $\frac{1 + i}{2}$ (c) $\frac{1}{2} + i$ (d) $1 + \frac{i}{2}$ (e) $\frac{1 + i\sqrt{13}}{2}$

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PROBLEM 5 The sum of 18 consecutive positive integers is a perfect square. What is the smallest possible value of this sum?

PROBLEM 6 For all positive integers n less than 2002, let

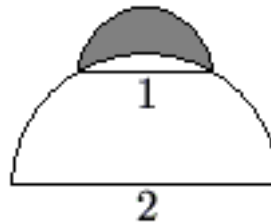
$$a_n = \begin{cases} 11, & \text{if } n \text{ is divisible by 13 and 14,} \\ 13, & \text{if } n \text{ is divisible by 14 and 11,} \\ 14, & \text{if } n \text{ is divisible by 11 and 13,} \\ 0, & \text{otherwise.} \end{cases}$$

Calculate

$$\sum_{n=1}^{2000} a_n.$$

PROBLEM 7 In $\triangle ABC$, we have $AB = 1$ and $AC = 2$. Side BC and the median from A to BC have the same length. How long is side BC ?

PROBLEM 8 A semicircle of diameter 1 sits at the top of a semicircle of diameter 2, as shown. The shaded area inside the smaller semicircle and outside the larger semicircle is called a lune. Determine the area of this lune.



PROBLEM 9 For which ordered pairs of real numbers b and c do both roots of the quadratic equation

$$z^2 + bz + c = 0$$

lie inside the unit disk $\{|z| < 1\}$ in the complex plane?

PROBLEM 10 Consider all lines which meet the graph of

$$y = 2x^4 + 7x^3 + 3x - 5$$

in four distinct points (x_i, y_i) , $i = 1, 2, 3, 4$. Show that

$$\frac{x_1 + x_2 + x_3 + x_4}{4}$$

is independent of the line and find its value.

Pizza and Problems

1 Wiki Page

Our wiki page for Pizza and Problems is located at the following URL:

http://msenux.redwoods.edu/wiki/index.php/Pizza_and_Problems

If interested in editing solutions on this page, you need an account. If you wish an account, post an email to david-arnold@redwoods.edu that includes a username and password which you wish to use to log into the wiki.

