

Pizza and Problems

Fall 2009

Assigned on: October 16, 2009

Due on: October 16, 2009

PROBLEM 1 The sum of the two 5-digit numbers AMC10 and AMC12 is 123422. What is $A + M + C$?

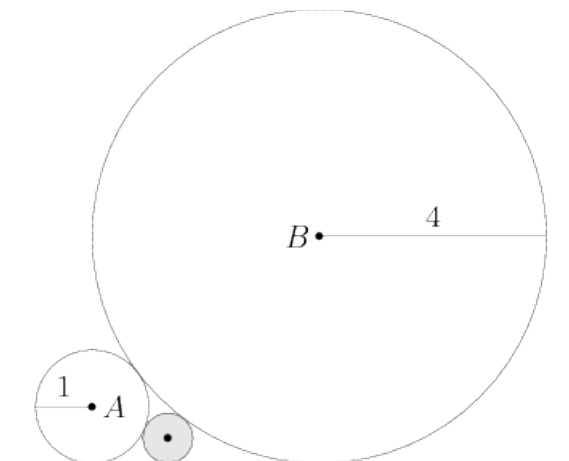
PROBLEM 2 For how many ordered pairs of positive integers (x, y) is $x + 2y = 100$?

PROBLEM 3 A point (x, y) is randomly picked from inside the rectangle with vertices $(0, 0)$, $(4, 0)$, $(4, 1)$, and $(0, 1)$. What is the probability that $x < y$?

PROBLEM 4 What is the units digit of 13^{2003} ?

PROBLEM 5 An object moves 8 cm in a straight line from A to B , turns at an angle α , measured in radians and chosen at random from the interval $(0, \pi)$, and moves 5 cm in a straight line to C . What is the probability that $AC < 7$?

PROBLEM 6 A circle centered at A with a radius of 1 and a circle centered at B with a radius of 4 are externally tangent. A third circle is tangent to the first two and to one of their common external tangents as shown. What is the radius of the third circle?



PROBLEM 7 Let f be a function with the following properties:

$$f(1) = 1$$

$$f(2n) = nf(n), \text{ for any positive integer } n$$

What is the value of $f(2^{100})$?

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PROBLEM 8 If

$$\sum_{n=0}^{\infty} \cos^{2n} \theta = 5,$$

what is the value of $\cos 2\theta$?

PROBLEM 9 Three mutually tangent spheres of radius 2 rest on a horizontal plane. A sphere of radius 2 rests on them. What is the distance from the plane to the top of the larger sphere?

PROBLEM 10 The set of all real numbers x for which

$$\log_{2004}(\log_{2003}(\log_{2002}(\log_{2001} x)))$$

is defined for $\{x : x > c\}$. What is the value of c ?

PROBLEM 11 In rectangle $ABCD$, we have $AB = 8$, $BC = 9$, H is on BC with $BH = 6$, E is on AD with $DE = 4$, line EC intersects line AH at G , and F is on the line AD with $GF \perp AF$. Find the length of GF .

1 Wiki Page

Our wiki page for Pizza and Problems is located at the following URL:

http://msenex.redwoods.edu/wiki/index.php/Pizza_and_Problems

If interested in editing solutions on this page, you need an account. If you wish an account, post an email to david-arnold@redwoods.edu that includes a username and password which you wish to use to log into the wiki.