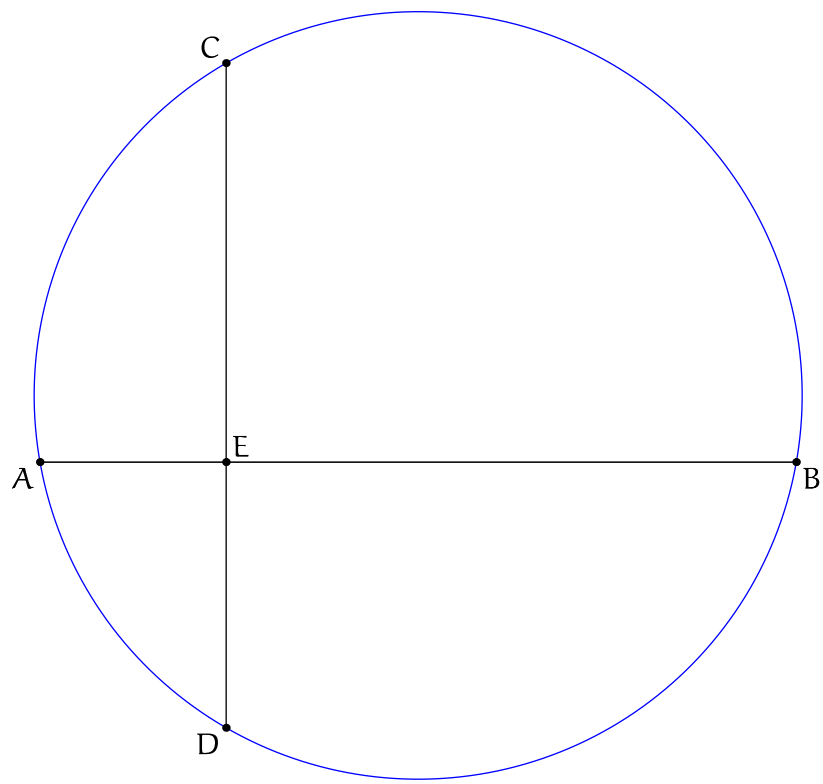
Pizza and Problems

Spring 2008

Assigned on: April 18, 2008 Due on: April 18, 2008

Problem 1 If $a \pm bi$ ($b \neq 0$, $i = \sqrt{-1}$) are imaginary roots of the the equation $x^2 + qx + r = 0$, where a, b, q, and r are real numbers, find q in terms of a and b.

Problem 2 Chords AB and CD in the circle that follows intersect at E and are prepedicular to each other. If segments AE, EB, and ED have measures 2, 6, and 3, respectively, find the length of the diameter of the circle.



Problem 3 Let C be the unit circle $x^2+y^2 = 1$. A point p is chosen randomly on the circumference of C and another point q is chosen randomly from the interior of C (these points are chosen independently and uniformly over their domains). Let R be the rectangle with sides parallel to the x and y-axes with diagonal pq. What is the probability that no point of R lies outside of C?

Problem 4 Three times Dick's age plus Tom's age equals twice Harry's age. Double the cube of Harry's age is equal to three times the cube of Dick's age added to the cube of Tom's age. Their respective ages are relatively prime to each other. What is the sum of the squares of their ages?

Problem 5 When the number 2^{1000} is divided by 13, what is the remainder?

PROBLEM 6 Find the greatest integer that will divide 13,511, 13,903, and 14,589 and leave the same remainder.

PROBLEM 7 Assuming that a match is a unit of length, it is possible to place 12 matches on a plane in various ways to form polygons with integral areas. How can you use all 12 matches (the entire length of each match must be used) to form a polygon with an area of exactly four square units?

PROBLEM 8 A rectangle is inscribed in the quadrant of a circle as shown in the figure that follows. Given the distances indicated, can you accurately determine the length of the diagonal AC?