

Pizza and Problems

Spring 2009

Assigned on: January 30, 2009

Due on: January 30, 2009

PROBLEM 1 Evaluate

$$\sqrt[8]{2207 - \frac{1}{2207 - \frac{1}{2207 - \dots}}}$$

Express your answer in the form

$$\frac{a + b\sqrt{c}}{d}, \quad (1)$$

where a , b , c , and d are integers.

PROBLEM 2 For how many positive integers n is $n^3 - 8n^2 + 20n - 13$ a prime number?

PROBLEM 3 Let f be a real-valued function such that

$$f(x) + 2f\left(\frac{2002}{x}\right) = 3x$$

for all $x > 0$. Find $f(2)$.

PROBLEM 4 The vertex E of square $EFGH$ is at the center of square $ABCD$. The length of a side of $ABCD$ is 1 and the length of a side of $EFGH$ is 2. Side EF intersects CD at I and EH intersects AD at J . If angle $EID = 60^\circ$, find the area of quadrilateral $EIDJ$.

PROBLEM 5 Consider the polynomials

$$p(x) = x^6 - x^5 - x^3 - x^2 - x$$

and

$$q(x) = x^4 - x^3 - x^2 - 1.$$

Given that z_1, z_2, z_3 , and z_4 are the roots of $q(x) = 0$, find

$$p(z_1) + p(z_2) + p(z_3) + p(z_4).$$

PROBLEM 6 In $\triangle ABC$, $AB = 13$, $BC = 14$, $AC = 15$, and point G is the intersection of the medians. Points A' , B' , and C' are the images of A , B , and C , respectively, after a 180° rotation about G . What is the area of the union of the two regions enclosed by the triangles ABC and $A'B'C'$?

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PROBLEM 7 A certain state issues license plates consisting of six digits (from 0 through 9). The state requires that any two plates differ in a least two places. (Thus, the plates $\boxed{027592}$ and $\boxed{020592}$ cannot both be used.) Determine, with proof, the maximum number of distinct license plates that the state can use.

PROBLEM 8 Let N be the greatest integer multiple of 8, no two of whose digits are the same. What is the remainder when N is divided by 1000?

1 Wiki Page

Our wiki page for Pizza and Problems is located at the following URL:

http://msenux.redwoods.edu/wiki/index.php/Pizza_and_Problems

If interested in editing solutions on this page, you need an account. If you wish an account, post an email to david-arnold@redwoods.edu that includes a username and password which you wish to use to log into the wiki.
