Pizza and Problems

Spring 2009

Assigned on: February 20, 2009 Due on: February 20, 2009

Problem 1 Let $ABCD$ be a cyclic quadrilateral (a quadrilateral which can be inscribed in a circle). Let E and F be variable points on the sides AB and CD, respectively, such that $AE/EB = CF/FD$. Let P be the point on the segment EF such that $PE/PF = AB/CD$. Prove that the ratio between the areas of triangle APD and BPC does not depend on the choice of E and F.

Problem 2 Determine the area of the region defined by the inequality

$$|3x - 18| + |2y + 7| \leqslant 3.$$

Problem 3 Find the tens digit in the sum

$$7! + 8! + 9! + \cdots + 2006!.$$

Problem 4 In the following equation, each letter represents uniquely a different digit in base ten:

$$(YE) \cdot (ME) = TTT$$

The sum $E + M + T + Y$ equals what number?

Problem 5 Find the area of rhombus $ABCD$ given that the radii of the circles circumscribed around triangles ABD and ACD are 12.5 and 25, respectively.

Problem 6 The Dunbar family consists of a mother, a father, and some children. The average age of the members of the family is 20, the father is 48 years old, and the average age of the mother and children is 16. How many children are in the family?

Problem 7 How many real solutions does the equation

$$\frac{x}{100} = \sin x$$

have?

Problem 8 How many pairs of positive integers (a, b) are there such that a and b have no common factors greater than 1 and

$$\frac{a}{b} + \frac{14b}{9a}$$

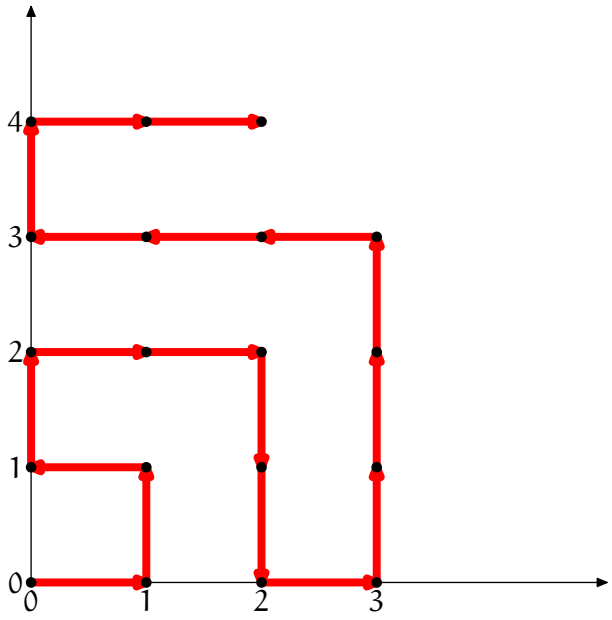
is an integer?

PROBLEM 9 Let a, b, c, d, and e be distinct integers such that

$$(6-a)(6-b)(6-c)(6-d)(6-e) = 45.$$

What is $a+b+c+d+e$?

PROBLEM 10 A particle moves through the first quadrant as follows. During the first minute it moves from the origin to $(1,0)$. Thereafter, it continues to follow the directions indicated in the figure, going back and forth between the positive x and y axes, moving one unit of distance parallel to an axis in each minute. At which point will the particle be after exactly 1989 minutes?



1 Wiki Page

Our wiki page for Pizza and Problems is located at the following URL:

http://msenux.redwoods.edu/wiki/index.php/Pizza_and_Problems

If interested in editing solutions on this page, you need an account. If you wish an account, post an email to david-arnold@redwoods.edu that includes a username and password which you wish to use to log into the wiki.