

Pizza and Problems

Spring 2009

Assigned on: March 27, 2009

Due on: March 27, 2009

PROBLEM 1 The first two terms of a sequence are $a_1 = 1$ and $a_2 = 1/\sqrt{3}$. For $n \geq 14$,

$$a_{n+1} = \frac{a_n + a_{n+1}}{1 - a_n a_{n+1}}.$$

What is $|a_{2009}|$?

PROBLEM 2 Convex quadrilateral ABCD has $AB = 9$ and $CD = 12$. Diagonals AC and BD intersect at E, $AC = 14$, and $\triangle AED$ and $\triangle BEC$ have equal areas. What is AE?

PROBLEM 3 Andrea inscribed a circle inside a regular pentagon, circumscribed a circle around the pentagon, and calculated the area of the region between the two circles. Bethany did the same with a regular heptagon (7 sides). The areas of the two regions were A and B, respectively. Each polygon had a side length of 2. Which of the following is true?

- (a) $A = \frac{25}{49}B$ (b) $A = \frac{5}{7}B$ (c) $A = B$ (d) $A = \frac{7}{5}B$ (e) $A = \frac{49}{25}B$

PROBLEM 4 For what value of n is $i + 2i^2 + 3i^3 + \dots + ni^n = 48 + 49i$?

PROBLEM 5 How many positive integers less than 1000 are 6 times the sum of their digits?

PROBLEM 6 For how many values of x in $[0, \pi]$ is $\sin^{-1}(\sin 6x) = \cos^{-1}(\cos(x))$?

PROBLEM 7 A region S in the complex plane is defined by

$$S = \{x + iy : -1 \leq x \leq 1, -1 \leq y \leq 1\}.$$

A complex number $z = x + iy$ is chosen uniformly at random from S. What is the probability that

$$\left(\frac{3}{4} + \frac{3}{4}i\right)z$$

is also in S?

PROBLEM 8 The solutions of $z^4 + 4z^3i - 6z^2 - 4zi - i = 0$ are the vertices of a convex polygon in the complex plane. What is the area of the polygon?

- (a) $2^{5/8}$ (b) $2^{3/4}$ (c) 2 (d) $2^{5/4}$ (e) $2^{3/2}$

PROBLEM 9 Let $k = 2008^2 + 2^{2008}$. What is the units digit of $k^2 + 2^k$?

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PROBLEM 10 The numbers $\log(a^3b^7)$, $\log(a^5b^{12})$, and $\log(a^8b^{15})$ are the first three terms of an arithmetic sequence, and the 12th term of the sequence is $\log(b^n)$. What is n ?

- (a) 40 (b) 56 (c) 76 (d) 112 (e) 143

1 Wiki Page

Our wiki page for Pizza and Problems is located at the following URL:

http://msenux.redwoods.edu/wiki/index.php/Pizza_and_Problems

If you are interested in editing solutions on this page, you need an account. If you wish an account, post an email to david-arnold@redwoods.edu that includes a username and password which you wish to use to log into the wiki.
