Pizza and Problems

Spring 2009

Assigned on: April 10, 2009 Due on: April 10, 2009

Problem 1 Triangles $\triangle ABC$ and $\triangle ADC$ are isosceles with $AB = BC$ and $AD = DC$. Point D is inside triangle $\triangle ABC$, angle $\angle ABC$ measures 40 degrees, and angle $\angle ADC$ measures 140 degrees. What is the degree measure of angle $\angle BAD$?

Solution 50

Problem 2 A star-polygon is drawn on a clock face by drawing a chord from each number to the fifth number counted clockwise from that number. That is, chords are drawn from 12 to 5, from 5 to 10, from 10 to 3, and so on, ending back at 12. What is the degree measure of the angle at each vertex in the star polygon?

Solution 30

Problem 3 Integers a, b, c, and d, not necessarily distinct, are chosen independently and at random form 0 to 2007, inclusive. What is the probability that $ad - bc$ is even?

Solution 5/8

Problem 4 A piece of cheese is located at $(12, 10)$ in the coordinate plane. A mouse is at $(4, -2)$ and is running up the line $y = -5x + 18$. At the point (a, b) the mouse starts getting farther from the cheese rather than closer to it. What is $a + b$?

Solution 10

Problem 5 How many three-digit numbers are composed of three distinct digits such that one digit is the average of the other two?

Solution 112

Problem 6 For each positive integer n, let $S(n)$ denote the sum of the digits of n. For how many values of n is $n + S(n) + S(S(n)) = 2007$?

Solution 4

PROBLEM 7 Square ABCD has area 36, and segment $\overline{AB}$ is parallel to the x-axis. Vertices A, B, and C are on the graphs of $y = \log_a x$, $y = 2\log_a x$, and $y = 3\log_a x$, respectively. What is a?

Solution $\sqrt[6]{3}$

PROBLEM 8 Two particles move along the edges of equilateral triangle $\triangle ABC$ in the direction

$$A \to B \to C \to A,$$

starting simultaneously and moving with the same speed. One starts at A, and the other starts at the midpoint of segment $\overline{BC}$. The midpoint of the line segment joining the two particles traces out a path that encloses a region R. What is the ratio of the area of region R to the area of the triangle $\triangle ABC$?

Solution 1/16

PROBLEM 9 Call a set of integers *spacy* if it contains no more than one out of any three consecutive integers. How many subsets of $\{1, 2, 3, \ldots, 12\}$ including the empty set, are spacy?

Solution 129

PROBLEM 10 For each integer $n > 1$, let $F(n)$ be the number of solutions to the equation $\sin x = \sin(nx)$ on the interval $[0, \pi]$. What is $\sum_{n=2}^{2007} F(n)$?

Solution 2016532

1 Wiki Page

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