

Pizza and Problems

Spring 2009

Assigned on: April 24, 2009

Due on: April 24, 2009

PROBLEM 1 What is:

$$(-1)^1 + (-1)^2 + \cdots + (-1)^{2006}$$

Solution 0

PROBLEM 2 How many even three-digit integers have the property that their digits, read left to right, are in strictly increasing order?

Solution 34

PROBLEM 3 Square EFGH is inside the square ABCD so that each side of EFGH can be extended to pass through a vertex of ABCD. Square ABCD has side length $\sqrt{50}$ and $BE = 1$. What is the area of the inner square EFGH?

Solution 36

PROBLEM 4 A faulty car odometer proceeds from digit 3 to digit 5, always skipping the digit 4, regardless of position. If the odometer now reads 002005, how many miles has the car actually traveled?

Solution 1462

PROBLEM 5 For how many real values of x is $\sqrt{120 - \sqrt{x}}$ an integer?

Solution 11

PROBLEM 6 Circles with centers $(2, 4)$ and $(14, 9)$ have radii 4 and 9, respectively. The equation of a common external tangent to the circles can be written in the form $y = mx + b$ with $m > 0$. What is b ?

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Solution 912/119

PROBLEM 7 Let

$$S_1 = \{(x, y) : \log_{10}(1 + x^2 + y^2) \leq 1 + \log_{10}(x + y)\}$$

and

$$S_2 = \{(x, y) : \log_{10}(2 + x^2 + y^2) \leq 2 + \log_{10}(x + y)\}.$$

What is the ratio of the area of S_2 to the area of S_1 ?

Solution 102

PROBLEM 8 A circle of radius r is concentric with and outside a regular hexagon of side length 2. The probability that three entire sides of the hexagon are visible from a randomly chosen point on the circle is $1/2$. What is r ?

Solution $3\sqrt{2} + \sqrt{6}$

PROBLEM 9 Let S be the set of all points (x, y) in the coordinate plane such that $0 \leq x \leq \pi/2$ and $0 \leq y \leq \pi/2$. What is the area of the subset of S for which

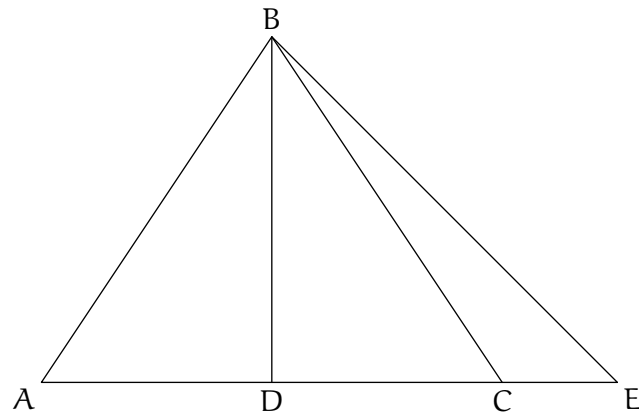
$$\sin^2 x - \sin x \sin y + \sin^2 y \leq \frac{3}{4}.$$

Solution $\pi^2/6$

PROBLEM 10 Rectangle $ABCD$ has area 2006. An ellipse with area 2006π passes through A and C and has foci at B and D . What is the perimeter of the rectangle? (*Hint: The area of an ellipse is πab where $2a$ and $2b$ are the lengths of its axes.*)

Solution $8\sqrt{1003}$

PROBLEM 11 In $\triangle ABC$, $AB = BC$, and \overline{BD} is an altitude. Point E is on the extension of \overline{AC} such that $BE = 10$. The values of $\tan \angle CBE$, $\tan \angle DBE$, and $\tan \angle ABE$ form a geometric progression, and the values of $\cot \angle DBE$, $\cot \angle CBE$, and $\cot \angle DBC$ form an arithmetic progression. What is the area of $\triangle ABC$?



Solution 50/3

1 Wiki Page

Our wiki page for Pizza and Problems is located at the following URL:

http://msenux.redwoods.edu/wiki/index.php/Pizza_and_Problems

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